97

BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT

FINAL EXAM **MATH 235** **Spring 2011**

Name ..Rawan..Ahmad.................

Number.1041163

Sec..(5)..... محاسبة

PART I: Multiple choices questions & true false questions

Question # 1 (48 %): Circle the correct answer

(1) The equation $4x(x-1)=3x^2-4$

$4x^2-4x-3x^2+4=0$
$x^2-4x+4=0$
$(x-2)(x-2)$
$x=2$

(a) Has no solution
(b) Has one solution
(c) Has two solutions
(d) Has {0} as a solution

(2) Write an equation in standard form for the line through (3,-5) and (-3,7)

$x_1\ y_1\quad x_2\ y_2$

(a) $2x+y=11$
(b) $x+2y=1$
(c) $x+2y=-7$
(d) $2x+y=1$

$\frac{y_2-y_1}{x_2-x_1}=\frac{7+5}{-3-3}=\frac{12}{-6}=-\frac{1}{2}$

$-5+\frac{3}{2}$

$y-y_1=m(x-x_1)$
$y+5=-\frac{1}{2}x+\frac{3}{2}$
$y=-\frac{1}{2}x-\frac{7}{2}$
$y+\frac{1}{2}x=-\frac{7}{2}$
$2y+x=-7$

(3) Suppose that the variable cost of producing an item is \$500 and the fixed cost is \$200. Find a linear cost function for production of this item

$500x+200$

(a) $C(x)=200x+300$
(b) $C(x)=300x+200$
(c) $C(x)=500x+200$
(d) $C(x)=300x$

(4) If $A=\begin{bmatrix}2&3\\3&5\end{bmatrix}$, then $A^{-1}=$ $\frac{1}{ad-bc}=\frac{1}{10-9}\begin{bmatrix}5&-3\\-3&2\end{bmatrix}$

(a) $\begin{bmatrix}5&-3\\-3&2\end{bmatrix}$

(b) $\begin{bmatrix}5&3\\3&2\end{bmatrix}$

(c) $\begin{bmatrix}2&3\\3&5\end{bmatrix}$

(d) None of the above

$x_1\ y_1 \quad x_2\ y_2$

(5) The slope of the line passing through the points (1,-2) and (3,4) is

(a) 0.5
(b) -2
(c) 3
(d) 1

$\frac{y_2-y_1}{x_2-x_1} = \frac{4+2}{3-1} = \frac{6}{2} = 3$

(6) Which of the following can be a transition matrix for a Markov Chain

(a) $\begin{bmatrix} 0.75 & 0.25 \\ 0.35 & 0.65 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & -4 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 0.6 & 0.4 \\ 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix}$

(d) $\begin{bmatrix} 0.68 & 0.41 \\ 0.32 & 0.59 \end{bmatrix}$

(7) Find x such that $\log_8(x) = \frac{-5}{3}$

(a) $\frac{1}{32}$

(b) $\frac{1}{8}$

(c) $\frac{1}{16}$

(d) None of the above

$(\sqrt[3]{8})^{-5} = x$

$2^{-5} = x$

$\frac{1}{2^5} = x$

(8) If the supply function for a commodity is $p = q^2 + 8q + 16$ and the supply is $p = -3q^2 + 6q + 436$, find the equilibrium point

(a) (196,10)
(b) (10,10)
(c) (10,196)
(d) (10,21)

$100 + 80 + 16$
196

$q^2 + 8q + 16 = -3q^2 + 6q + 436$

$4q^2 + 2q - 420 = 0$

$2q^2 + q - 210 = 0$

$(2x + 10)(x + 21)$

$2x \cdot -10x$

$-20x$
$+21x$
$1x$

$x = 10$

(9) If $f(x) = (9 - x^2)^{\frac{2}{3}}$, then $f'(x) =$

(a) $\frac{-4x}{3(9-x^2)^{\frac{1}{3}}}$

(b) $\frac{4x}{3(9-x^2)^{\frac{1}{3}}}$

(c) $\frac{4x}{3}$

(d) None of the above

$\frac{2}{3}(9-x^2)^{-\frac{1}{3}} \cdot -2x$

$\frac{-4x}{(9-x^2)^{\frac{1}{3}}}$

$\frac{1}{2}$

(10) Suppose the demand function is $p(x) = \frac{100}{\sqrt{x}}$ and the cost function is $C(x) = x + 500$. Find the marginal profit when x = 2500

(a) 2000
(b) -1950
(c) 1
(d) 0

$\frac{\sqrt{x}\,100 - \frac{1}{2}x^{-\frac{1}{2}}\,100x}{x}$

$\frac{5000 - \frac{x}{100}\,2500}{2500}$

$\frac{100x}{\sqrt{x}} - x - 500$

$\frac{100x}{x^{\frac{1}{2}}} = 100x^{\frac{1}{2}} - x - 500$

$\frac{50}{\sqrt{x}} - 1$

$\frac{50}{50}$ 0

(11) $xe^x + x = e^y$, find $\frac{dy}{dx}$

(a) $\frac{x\ e^x + x + 1}{e^y}$

(b) $\frac{e^x}{e^y}$

(c) $\frac{x\ e^x + e^x}{e^y}$

(d) $\frac{e^x}{ye^y + e^y}$

$xe^x + e^x + 1 = e^y y'$

$y' = \frac{xe^x + e^x + 1}{e^y}$

(12) If $f(x) = 5x^4 - x^5 + 10$, then f(x) has

(a) a relative min at x=0 and a relative max at x=4
(b) a relative max at x =0
(c) a relative min at x= 4
(d) a relative max at x=0 and a relative min at x=4

$20x^3 - 5x^4$

$4x^3 - x^4 = 0$

$x^3(4 - x)$

$x = 0$
$x = 4$

$60x^2 - 20x^3$

$60(16) - 20(32)$

$960 = 320$

(13) If $f_x(1,0) = f_y(1,0) = 0$, and $f_{xx}(1,0) = -3, f_{xy}(1,0) = 2, f_{yy}(1,0) = -2$, then (1,0) is

(a) Maximum point
(b) Minimum point
(c) Neither maximum nor minimum
(d) We can't tell

$D = f_{xx}f_{yy} - (f_{xy})^2$

$6 - 4 = 2$

f_{xx}

(14) Which of the following statements are true?

I. ln (a+b) = ln (a) ln (b)
II. ln (cx) =c ln(x) ,where c is a positive constant

(a) I only
(b) II only
(c) Neither I nor II
(d) I and II

(15) If $f(x,y) = 4x^3 + 2x^2y + y^2$, then $f_{xx}(3,-1) =$ $f_x = 12x^2 + 4xy$

(a) 96

(b) 16 $f_{xx} = 24x + 4y$

(c) 68

(d) None of the above $24(3) - 4$

1/2

(16) If $f(x) = 2^{2x+2}$, then $f'(0) =$ 2^{2x+2} $\frac{1}{\ln 2}$ 2

(a) 16

(b) ln 16

(c) 2 ln 16

(d) None of the above $\frac{8}{\ln 2}$ $2^{2x+2} \ln 2 \cdot 2$

$8 \ln 2$

$2 \ln 16$ $2 \times 4 \ln 2$

$2 \ln 2^4$

Question # 2 True or False (10%):

(1) ---False--- If A, B are two $n \times n$ matrices such that AB = 0, then A = 0 or B=0

(2) ---False--- If [u v w] is the initial vector for a Markov Chain then u + v + w =0

(3) ---True--- The function $f(x) = x^3 - 3x^2$ has a maximum value at x=0

$3x^2 - 6x$ $x^2 - 2x = 0$

$6x - 6$ $x(x-2) = 0$

$x = 0$

(4) ---~~True~~ False--- If $f(x) = e^{3x+2}$, then $f'(0) = 3e^2$ $x = 2$

$e^{3x+2}\ 3$ e^2

(5) ---False---If $f_x(a,b) = f_y(a,b) = 0$, then f(x,y) has a relative maximum at (a, b)

8

PART 2: SHOW ALL YOUR WORK

Question # 1(15%)

Find y' for the following. You need not simplify

(a) $f(x)=\dfrac{5}{(2x^2+4)^7}$

$f(x)=5(2x^2+4)^{-7}$

$y'=-35(2x^2+4)^{-8}\cdot 4x$

$y'=-140x(2x^2+4)^{-8} = \dfrac{-140x}{(2x^2+4)^8}$

(b) $f(x)=x^3+\dfrac{2}{x}+\sqrt{x}+2x$

$y'=3x^2-\dfrac{2}{x^2}+\dfrac{1}{2\sqrt{x}}+2$

$y'=3x^2-\dfrac{2}{x^2}+\dfrac{1}{2\sqrt{x}}+2$

5

(c) $y\ln x+x^3=y^2+2$

$y\frac{1}{x}+y'\ln x+3x^2=2yy'$

$\frac{y}{x}+3x^2=2yy'-y'\ln x$

$\frac{y}{x}+3x^2=y'(2y-\ln x) \Rightarrow y'=\dfrac{\frac{y}{x}+3x^2}{2y-\ln x}$

$2x+4)^{\frac{1}{2}}$

(d) $f(x)=5^{\sqrt{2x+4}}$

$y'=5^{\sqrt{2x+4}}\ln 5\ \frac{1}{2}(2x+4)^{-\frac{1}{2}}$

$=5^{\sqrt{2x+4}}\ln 5\ \dfrac{2}{2\sqrt{2x+4}} = \dfrac{5^{\sqrt{2x+4}}\ln 5}{\cancel{2}\sqrt{2x+4}}$

(e) $f(x)=\ln\sqrt{x^3+10}$

$f(x)=\ln(x^3+10)^{\frac{1}{2}}$

$\frac{1}{2}\ln(x^3+10)$

$=\dfrac{1}{2}\ \dfrac{3x^2}{x^3+10}$

$= 3x^2$

Question # 3 (12%)

a. Suppose you invest \$8000 in each of 2 banks. The first compounds quarterly at an annual rate of 10% and the second compounds continuously at 9%. At the end of 5 years which has more money in it.

4

First bank

$A = P\left(1+\frac{r}{m}\right)^{mn}$

$= 8000\left(1+\frac{.1}{4}\right)^{4(5)}$

$= 8000(1.64)$

$= \boxed{13109}$ the First bank has more money in it.

Second bank

$A = Pe^{rt}$

$A = 8000e^{.09(5)}$

$= 8000e^{.45}$

$= 8000(1.6)$

$= 12546$

b. If the total cost function is given by $C(x) = x^2 + 12x + 36$. Find the minimum average cost

12

$C(x) = x^2 + 12x + 36$

$\overline{C(x)} = \frac{x^2 + 12x + 36}{x}$

$= x + 12 + \frac{36}{x}$

$\overline{C(x)}' = 1 - \frac{36}{x^2}$

critical value (# of unit)

$1 - \frac{36}{x^2} = 0 \Rightarrow 1 = \frac{36}{x^2} \Rightarrow x^2 = 36$

$x = \sqrt{36}$

$x = \boxed{6}$

-6 (No # of unit in minus)

$-36x^{-2}$

$72x^{-3}$

$\overline{C(x)}'' = \frac{72}{x^3}$

$\frac{72}{(6)^3} > 0$ has minimum value

$\overline{C(6)} = 6 + 12 + 36$

Question # 3 (20%)
A company produces two products at a total cost

$$C(x,y) = x^2 + 200x + y^2 + 100y - xy$$

where x and y represent the units produced of each product. The revenue function is

$$R(x,y) = 2000x - 2x^2 - y^2 + 100y + xy$$

i. **Find the marginal cost with respect to y at (10,20)**
ii. **Find the marginal profit with respect to x at (10,20)**
iii. **Find the number of units of each product that will maximize profit**

$2000x - 2x^2 - y^2 + 100y +$
$+ \; -200x - x^2 - y^2 - 100y +$

$1800x - 3x^2 - 2y^2 + 2xy$

i. $C(x,y) = x^2 + 200x + y^2 + 100y - xy$

$C_y = 2y + 100 - x$

$= 2(20) + 100 - 10$

$= 40 + 100 - 10 = 130$

ii. $P(x,y) = R(x) - C(x)$

$= 2000x - 2x^2 - y^2 + 100y + xy - x^2 - 200x - y^2 - 100y + xy$

$P(x,y) = 1800x - 3x^2 - 2y^2 + 2xy$

$P_x = 1800 - 6x + 2y$

$= 1800 - 6(10) + 2(20)$

$= 1800 - 60 + 40$

$= 1780$

تابع →

iii. $P(x,y) = 1800x - 3x^2 - 2y^2 + 2xy$

$P_x = 1800 - 6x + 2y$

$P_{xx} = -6$

$P_y = -4y + 2x$

$P_{yy} = -4$

$P_{xy} = 2$

$P_x = 0 \quad , P_y = 0$

$1800 - 6x + 2y = 0$

$3/ -4y + 2x = 0$

$1800 - 6x + 2y = 0$

$-12y + 6x = 0$

$1800 - 10y = 0$

$2/ 1800 - 6x + 2y = 0$

$-4y + 2x = 0$

$3600 - 12x + 4y = 0$

$2x - 4y = 0$

$3600 - 10x = 0$

$3600 = 10x$